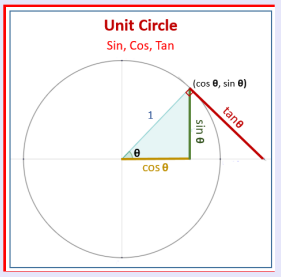
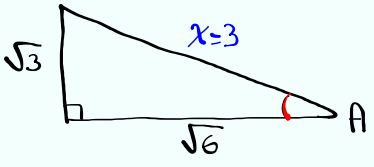
Trigonometry

Lecture 8



Feb 19-8:47 AM

Some Review



$(\sqrt{3})^2 + (\sqrt{6})^2 = x^2$

$3 + 6 = x^2$

$x^2 = 9$ $\quad x = 3$

$\frac{\sqrt{3}}{\sqrt{6}} = \frac{1\sqrt{3}}{\sqrt{2}\sqrt{3}} = \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Find

$\sin A = \frac{\sqrt{3}}{3}$ $\quad \csc A = \sqrt{3}$

$\cos A = \frac{\sqrt{6}}{3}$ $\quad \sec A = \frac{\sqrt{6}}{2}$

$\tan A = \frac{\sqrt{2}}{2}$ $\quad \cot A = \sqrt{2}$

$\frac{\sqrt{3}}{3} \rightarrow \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{6}}{3} \rightarrow \frac{3}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}}$

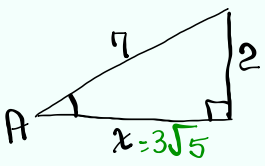
$\sqrt{36}$

$\frac{6}{2}$

Sep 9-10:27 AM

$\sin A = \frac{2}{7}$ in right-triangle ABC with $\angle C = 90^\circ$

1) Draw & clearly label.



$x^2 + 2^2 = 7^2$

$x^2 + 4 = 49$

$x^2 = 45$

$x = \sqrt{45}$

$= \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$

$\frac{7}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{15}$

$\frac{2}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{15}$

2) Find

$\sin A = \frac{2}{7}$ $\quad \csc A = \frac{7}{2}$

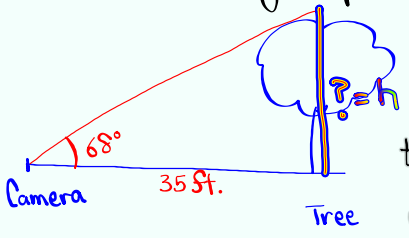
$\cos A = \frac{3\sqrt{5}}{7}$ $\quad \sec A = \frac{7\sqrt{5}}{15}$

$\tan A = \frac{2\sqrt{5}}{15}$ $\quad \cot A = \frac{3\sqrt{5}}{2}$

Sep 9-10:34 AM

A Camera on the ground is 35 ft from a tree.

Angle of elevation to the top of the tree from camera is 68°. Find the height of the tree. Drawing Required.



$\tan 68^\circ = \frac{h}{35}$

Cross-Multiply

$h = 35 \cdot \tan 68^\circ$

$h = 86.628$

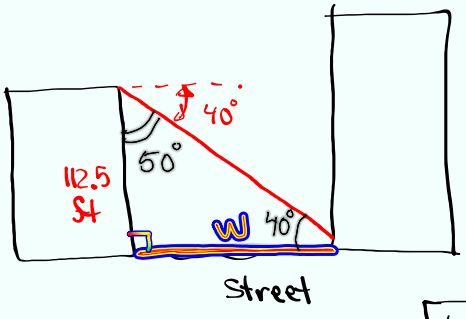
$h \approx \boxed{87 \text{ ft}}$

Sep 9-10:42 AM

A building is 112.5 ft tall.

From the roof, the angle of depression to the bottom of another building on the other side of the street is 40°.

How wide is the street? Drawing Required



$\tan 40° = \frac{112.5}{w}$

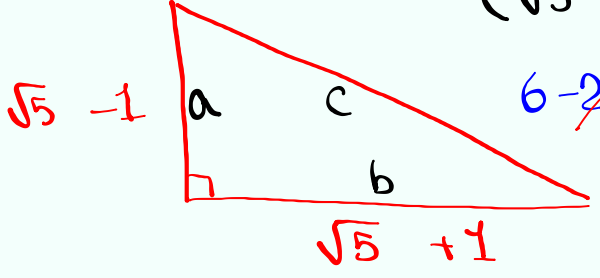
$w \tan 40° = 112.5$

$w = \frac{112.5}{\tan 40°}$

$w \approx 134.1$ ft

Sep 9-10:48 AM

Find the hypotenuse



$(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2 = c^2$

$6 - 2\sqrt{5} + 6 + 2\sqrt{5} = c^2$

$c^2 = 12$

$c = \sqrt{12} = \sqrt{4}\sqrt{3}$

$(\sqrt{5}-1)^2 = (\sqrt{5}-1)(\sqrt{5}-1) = \sqrt{25} - \sqrt{5} - \sqrt{5} + 1$

$= 6 - 2\sqrt{5}$

$c = 2\sqrt{3}$

Sep 9-10:54 AM

Rationalize the denominator

$$\frac{5}{\sqrt{7}-\sqrt{2}} \cdot \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{5(\sqrt{7}+\sqrt{2})}{\sqrt{49}+\sqrt{14}-\sqrt{14}-\sqrt{4}}$$

$$7 - 2$$

$$5$$

$$= \sqrt{7}+\sqrt{2}$$

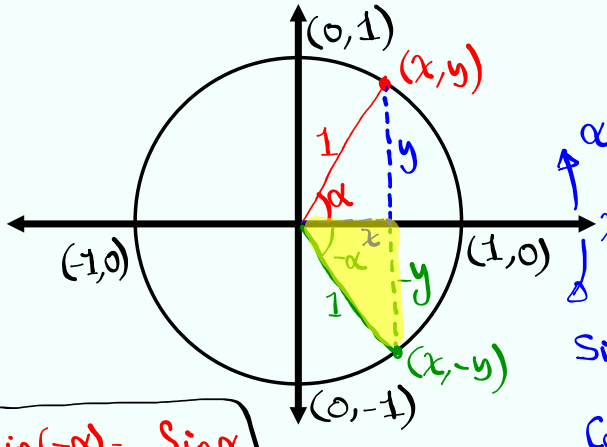
$$\frac{\sqrt{3}}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{\sqrt{9}-\sqrt{3}}{\sqrt{9}-\sqrt{3}+\sqrt{3}-1} = \frac{3-\sqrt{3}}{2}$$

$$3 - 1$$

Sep 9-10:59 AM

Consider the unit circle below:



$\sin\alpha = y$

$\cos\alpha = x$

$\alpha > 0$

$\tan\alpha = \frac{y}{x}$

$\alpha < 0$

$\sin(-\alpha) = -y$

$\cos(-\alpha) = x$

$\tan(-\alpha) = -\frac{y}{x}$

$\sin(-\alpha) = -\sin\alpha$

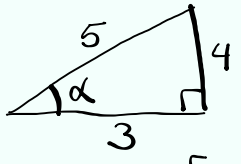
$\cos(-\alpha) = \cos\alpha$

$\tan(-\alpha) = -\tan\alpha$

Sep 9-11:06 AM

Given $\cos\alpha = \frac{3}{5}$ $\quad 0° < \alpha < 90°$

Find



$\sin(-\alpha) = -\sin\alpha = -\frac{4}{5}$

$\cos(-\alpha) = \frac{3}{5}$

$\tan(-\alpha) = -\tan\alpha = -\frac{4}{3}$

$\csc(-\alpha) = -\frac{5}{4}$

$\sec(-\alpha) = \frac{5}{3}$

$\cot(-\alpha) = -\frac{3}{4}$

Sep 9-11:14 AM

Simplify

$$(1 + \cot\alpha)^2 - \csc^2\alpha$$

$$= (1 + \cot\alpha)(1 + \cot\alpha) - \csc^2\alpha$$

$$= 1 + \cot\alpha + \cot\alpha + \cot^2\alpha - \csc^2\alpha$$

$$= 2\cot\alpha + \csc^2\alpha - \csc^2\alpha$$

$$= \boxed{2\cot\alpha}$$

Sep 9-11:19 AM

Simplify

$$\frac{1}{1+\sin\alpha}+\frac{1}{1-\sin\alpha}$$

$$=\frac{1(1-\sin\alpha)}{(1+\sin\alpha)(1-\sin\alpha)}+\frac{1(1+\sin\alpha)}{(1-\sin\alpha)(1+\sin\alpha)}$$

$$=\frac{1-\sin\alpha+1+\sin\alpha}{(1+\sin\alpha)(1-\sin\alpha)}$$

$$(1+\sin\alpha)(1-\sin\alpha)=1-\sin^2\alpha$$

$$=\frac{2}{\sin^2\alpha+\cos^2\alpha-\sin^2\alpha}=\frac{2}{\cos^2\alpha}=2\cdot\left(\frac{1}{\cos\alpha}\right)^2$$

$$=\boxed{2\sec^2\alpha}$$

Sep 9-11:24 AM

Distance formula between (x_1,y_1) & (x_2,y_2)

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Find distance between $(3,2)$ & $(-3,10)$

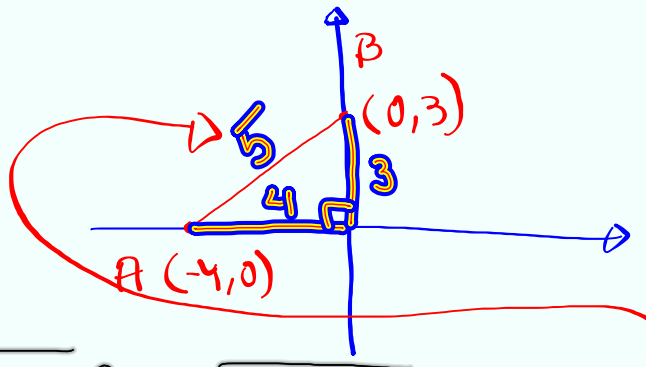
$$d=\sqrt{(3-{-3})^2+(2-10)^2}$$

$$=\sqrt{6^2+(-8)^2}=\sqrt{36+64}=\sqrt{100}=\boxed{10}$$

Sep 9-11:30 AM

Plot A(-4,0) and B(0,3), then

Find d(A,B).

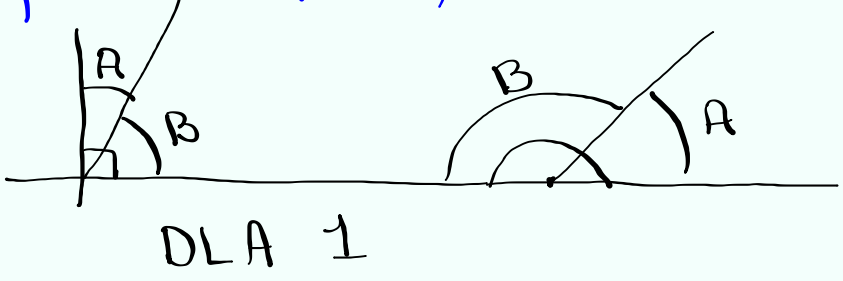


$$d = \sqrt{(-4-0)^2+(0-3)^2} = \sqrt{(-4)^2+(-3)^2} = \sqrt{16+9} = \sqrt{25}$$
$$= \boxed{5}$$

Sep 9-11:34 AM

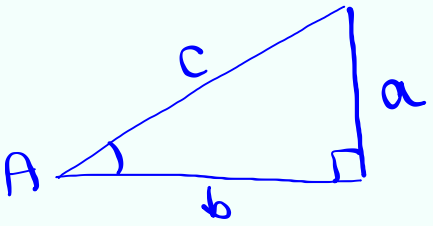
Complementary angles → $A+B=90^\circ$

Supplementary angles → $A+B=180^\circ$



DLA 1

Sep 9-11:38 AM



$$\sin^2 A + \cos^2 A =$$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 =$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2}$$

$$= \frac{c^2}{c^2} = 1$$

Sep 9-11:40 AM